

**Exercise 61**

Find an equation of the normal line to the curve  $y = \sqrt{x}$  that is parallel to the line  $2x + y = 1$ .

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**Solution**

Writing the given equation of the line as

$$y = -2x + 1,$$

we see that it has a slope of -2. The aim is to take the derivative of the given function and find where it's equal to the negative reciprocal,  $1/2$ .

$$\begin{aligned}y' &= \frac{d}{dx}(\sqrt{x}) \\ &= \frac{d}{dx}(x^{1/2}) \\ &= \frac{1}{2}x^{-1/2}\end{aligned}$$

Set this equal to  $1/2$  and solve for  $x$ .

$$\begin{aligned}\frac{1}{2}x^{-1/2} &= \frac{1}{2} \\ x^{-1/2} &= 1 \\ x &= 1\end{aligned}$$

Plug this value of  $x$  into the given function to get the corresponding  $y$ -value on the curve.

$$y(1) = \sqrt{1} = 1 \quad \Rightarrow \quad (1, 1)$$

Finally, determine the equation of the line with slope -2 that goes through the point  $(1, 1)$ .

$$y - 1 = -2(x - 1)$$

Below is a plot of the given curve, the given line, and this normal line parallel to the given line.

